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Extended warranty pricing in a competitive aftermarket under logit demand

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ABSTRACT

It is common for multiple firms—such as manufacturers, retailers, and third-party insurers—to coexist and compete in the aftermarket for durable products. In this paper, we study price competition in a partially concentrated aftermarket where one firm offers multiple extended warranty (EW) contracts while the others offer a single one. The demand for EWs is described by the multinomial logit model. We show that, at equilibrium, such an aftermarket behaves like a combination of monopoly and oligopoly. Building upon this base model, we further investigate sequential pricing games for a durable product and its EWs to accommodate the ancillary nature of after-sales services. We consider two scenarios: one where the manufacturer (as the market leader) sets product and EW prices *simultaneously*, and another where these decisions are made *sequentially*. Our analysis demonstrates that offering EWs incentivizes the manufacturer to lower the product price, thereby expanding the market potential for EWs. Simultaneous product–EW pricing leads to a price concession on EWs compared to sequential pricing, effectively reducing the intensity of competition in the aftermarket. Overall, the competitiveness of an EW hinges on its ability to deliver high value to consumers at low marginal cost to its provider. While our focus is on EWs, the proposed game-theoretical pricing models apply broadly to other ancillary after-sales services.

1. Introduction

In today's highly competitive marketplace, nearly all consumer durables come with base warranties. A base warranty is a contractual agreement between a manufacturer and its consumers, obligating the manufacturer to provide repairs, replacements, or refunds if the product fails to perform as intended within a prespecified time period after purchase (Blischke & Murthy, 1992). In addition to the manufacturer's base warranty, extended warranties (EWs) of various types are often available in the aftermarket for durable products, offering protection against product failures beyond the base warranty period. In general, an EW contract should specify at least three key terms: the protection period, the coverage (i.e., components or services covered or excluded), and the compensation policy upon failure (Murthy & Jack, 2014). Even though EWs have long been criticized for their excessively high profitability and low added values (see, e.g., Berner, 2004; Consumer Reports, 2014; UK Competition Commission, 2003), recent years have seen a steady expansion of the EW market. Notably, the global EW market size reached US$ 139.1 billion in 2023 and is projected to reach US$ 232.8 billion by 2032, indicating an annual growth rate of 5.8%.[1] The growth of the EW market is driven by consumer demand for robust protection against unforeseen repair costs, which can be largely explained by their probability distortion (i.e., consumers tend to overestimate product failure probability) and loss aversion behaviors (Abito & Salant, 2019; Jindal, 2015).

In reality, it is a common situation that multiple providers—such as manufacturers, retailers, and third-party insurers—coexist and compete in the EW market for durable products like consumer electronics, home appliances, and vehicles (UK Competition Commission, 2003). For example, the EW plans for iPhone devices can be purchased from the original manufacturer—Apple (i.e., AppleCare+), the retailer—for example, Best Buy (through its EW arm Geek Squad), or third-party providers—for example, SquareTrade, Upsie, and Asurion (Whitehead, 2023; Wiggers, 2017). Another example is Machinery Scope, which offers comprehensive EW solutions for various agricultural equipment sold by multiple original equipment manufacturers (e.g., John Deere, Case IH, New Holland).[2] In this sense, agricultural equipment owners could buy EW plans either from the original equipment manufacturers

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[1] https://www.imarcgroup.com/extended-warranty-market (accessed on July 20, 2024).
[2] https://machineryscope.com/extended-warranty-plans/ (accessed on July 20, 2024).

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(say, John Deere[3]) or from Machinery Scope. The aforementioned examples show that the EW market might be duopoly or oligopoly. In such a competitive environment, firms must anticipate their competitors' actions when making EW pricing decisions. Surprisingly, price competition in an EW market has been under-explored in the existing literature, as pointed out by Murthy (2016).

In this paper, we study price competition in a partially concentrated aftermarket, where n ($n \geq 2$) EW contracts are offered by k ($1 \leq k \leq n$) firms. Each of the first $k-1$ firms offers a single contract while the remaining firm provides $n-k+1$ contracts. Partial concentration is a fairly general structure that can characterize the real-world aftermarket for many durable products. For example, Wang et al. (2020) mention that an online store selling Huawei MateBook offers three EW options (see Figure 1 therein). If there are some other online/offline stores that only offer a single EW contract, then a partially concentrated aftermarket emerges. Moreover, partial concentration contains oligopoly (when $k = n$) and monopoly (when $k = 1$) as two special cases. In this work, consumer choices among the available EW contracts are described by the multinomial logit (MNL) choice model developed by McFadden (1974). The MNL model provides an analytically tractable and interpretable framework for capturing discrete choices among a finite set of differentiated alternatives, each characterized by a bundle of attributes (McFadden, 2001). The suitability of the MNL model for our analysis is supported both by the multi-attribute nature of after-sales services (e.g., EW contracts prescribe protection periods, coverages, compensation policies, deductibles, scheduled maintenance and beyond) and by empirical validation from prior studies (see, e.g., Chu & Chintagunta, 2009; Guajardo et al., 2016). Furthermore, rigorous econometric methods have long been established for MNL and its variants (Greene, 2017). By adopting the MNL modeling framework, our research not only derives high-level managerial insights but also provides decision support for EW pricing in competitive business environments.

We begin by studying a simultaneous pricing game where all firms in the partially concentrated aftermarket set their EW prices simultaneously. We show that, at equilibrium, the multi-contract firm's pricing policy exhibits an "equal-markup" feature that is commonly found in the monopoly setting, whereas the EW markup for each single-contract firm differs from each other. Since the price equilibria are implicit functions of themselves, an efficient computation method is developed to facilitate the firms' EW pricing decisions in competitive environments. We also conduct a comparison of equilibrium outcomes among partial concentration, monopoly, and oligopoly to examine the impact of market concentration. Our analysis reveals that market concentration softens competition. That is, when the aftermarket moves from oligopoly to partial concentration (or from partial concentration to monopoly), the EW prices of all firms become higher, leading to a reduction in the total purchase probability of EWs and consumer surplus. Moreover, we find that when the aftermarket moves from monopoly to oligopoly, the market shares of large-effective-attractiveness contracts would be redistributed to small-effective-attractiveness ones, reflecting an interesting *market-share redistribution* effect resulting from competition.

Since EWs are essentially ancillary after-sales services, consumers' EW purchase decisions are contingent on their prior purchase of—or at least commitment to purchase—the main product. To accommodate this ancillary nature, we further investigate price competition for the main product and its EWs where their prices are characterized by an equilibrium to a sequential game. Without loss of generality, we suppose that the manufacturer—in addition to being the sole product provider—offers multiple EW contracts. We consider two sequential product–EW pricing games, depending on whether the manufacturer, as the market leader, makes its product and EW pricing decisions simultaneously or sequentially (Heese, 2012; Wang et al., 2024). In this sense, the simultaneous EW pricing game discussed above serves as the second stage of the sequential games. Our analysis demonstrates that offering EWs incentivizes the manufacturer to lower the product price, thereby boosting product sales and thus raising the market potential for EWs. Compared with its sequential counterpart, simultaneous product–EW pricing leads to a price concession on EWs, thereby mitigating the intensity of price competition in the aftermarket.

Overall, our investigations reveal that the competitiveness of an EW provider lies in whether it can offer EW contract(s) with large effective-attractiveness. A larger effective attractiveness implies that the EW contract can offer a higher valuation to consumers at a lower marginal cost to the provider. Finally, it is worth noting that while our focus is specifically on EWs, the game-theoretical pricing models and the managerial insights of our work are largely applicable to other ancillary after-sales services, for example, lease and maintenance service contracts (Arts et al., 2025; Deprez et al., 2021; Hamidi et al., 2016; Murthy & Jack, 2014). The remainder of this paper is structured as follows. Section 2 briefly reviews the relevant literature. Section 3 formulates the pricing game in a partially concentrated aftermarket and characterizes the equilibrium. Two special market structures (i.e., oligopoly and monopoly) are also discussed, along with a comparison of equilibrium outcomes for the three market structures. Section 4 extends the analysis to sequential pricing games for the main product and EWs in which two game scenarios are considered. Section 5 concludes the paper. Additional analyses can be found in Appendix and all proofs are relegated to Online Supplement.

2. Literature review

Our work is closely related to the literature on price optimization and competition for EWs and, more generally, ancillary after-sales services. Most extant literature in this stream addresses price optimization for EWs from a single firm's perspective; see Bian et al. (2019), Gallego et al. (2014), Hartman and Laksana (2009), Jack and Murthy (2007), Lam and Lam (2001) and Wang and Ye (2021) for references. In particular, Wang et al. (2020) and Wang (2023) study the design and pricing of EW menus that consist of multiple differentiated options based on the MNL model. The optimal EW-menu pricing policies in these studies are of a common equal-markup form. The concentrated portion (i.e., the multi-contract firm) of our concerned aftermarket also adopts such an equal-markup pricing policy. On the other hand, some studies examine EW pricing from a multi-firm competition perspective. Heese (2012) and Jiang and Zhang (2011) investigate how a retailer's EW offering impacts manufacturers' base warranties in different supply chain environments. Both find that the retailer's EW generally exerts downward pressure on manufacturers' base warranties. Li et al. (2012) study optimal EW design (in terms of price and length) in supply chains, where an EW can be offered (or resold) either by the manufacturer or by the retailer. The work by Li et al. (2012) has been extended by, for example, Chai et al. (2021), He et al. (2018), Zhang et al. (2023) and Zhou and Wang (2022), who focus generally on which party—manufacturer, retailer, or third-party insurer—is better off by offering the EW while making optimal EW design decisions. Our work deviates from the aforementioned literature in that we focus on EW pricing in a partially concentrated aftermarket where multiple, instead of only one, firms provide EWs simultaneously.

Our price competition game is based on the MNL demand model that predicts how consumers choose among the EWs available in the aftermarket. In this sense, our work is also related to the extant literature on price competition under logit-type demand models. Early research in this stream focuses primarily on examining the existence and uniqueness of a Nash equilibrium (e.g., Anderson & De Palma, 2001; Anderson et al., 1992; Bernstein & Federgruen, 2004; Gallego et al., 2006). Interested audiences are referred to Federgruen and Hu

[3] https://www.deere.com/en/parts-and-service/warranty-and-protection-plans/extended-service-plans/ (accessed on July 20, 2024).

(2017) for an overview of price competition under logit-type demands. In particular, Gallego and Wang (2014) and Li and Huh (2011) address price optimization and competition under nested logit demand with asymmetric price sensitivities. The former work proves concavity of the total profit function with respect to market share and leverages this result to derive optimal solutions under monopoly and oligopoly. The latter shows that multi-product price competition reduces to a simpler log-supermodular game in which each firm determines a single "adjusted markup" as opposed to a complete price vector. Because MNL is a special case of nested logit, mathematically speaking, our simultaneous EW pricing game can be regarded as a special setting of those in Gallego and Wang (2014) and Li and Huh (2011). Loots and den Boer (2023) study competition and collusion with demand learning in a pricing duopoly under MNL demand. They show that the notation of collusion defined by joint-revenue maximization—analogous to the monopoly setting in our work—is not always beneficial to both firms compared to the Nash equilibrium. In addition, Wang et al. (2022) study a logit-based competition game, where firms compete on product price, quality level, and service duration, simultaneously. They find that the optimal quality level and service duration for each product can be determined independently of other products. Li and Webster (2024) investigate a risk-sensitive price competition game, where each firm maximizes a risk-adjusted (i.e., mean^2–variance) profit objective. An interesting finding is that at equilibrium some firms are driven to zero profit, in contrast to the positive-profit equilibrium in risk-neutral scenarios. In a recent work, Liu et al. (2025) study Bayesian Nash equilibrium in price competition under MNL demand where each firm's marginal cost is private information.

Building upon the extant price competition models under logit-type demands, our work focuses specifically on price competition in a partially concentrated aftermarket. We contribute to the literature by accommodating the ancillary nature of EWs and leveraging the simultaneous pricing game as a workhorse model to study sequential product–EW pricing games. In this regard, our work is also related to the few publications on retail strategies for durable products and ancillary EWs. Heese (2012) considers a supply chain where two manufacturers sell their products through the same retailer who also offers EWs. He shows that the retailer can often benefit from inducing simultaneous consideration of product and EW characteristics, rather than advertising the EW after consumers have selected a certain product. However, there is no competition in the aftermarket, and the product and EW prices are considered exogenous. Instead, Wang et al. (2024) examine the retail-strategy choice problem from an optimal pricing viewpoint. They consider a monopoly that offers an EW either simultaneously with or sequentially to the main product. A final work related to ours is that of Cohen and Whang (1997), who study competition in after-sales service quality and price between a manufacturer and an independent service provider. In their work, the manufacturer's decisions on service quality and price are made after the product pricing decision. We extend the studies of Cohen and Whang (1997) and Wang et al. (2024) in that (i) there is oligopolistic competition in EW prices, and (ii) both simultaneous and sequential retail strategies for the main product and its EWs are considered. Therefore, our model is capable of capturing the impact of competition on the interaction between optimal product–EW pricing decisions induced by simultaneous or sequential retail strategy.

3. Price competition in extended warranties

In this section, we study simultaneous price competition in a partially concentrated aftermarket. We characterize the demand for EWs in Section 3.1, formulate a price competition game and analyze its equilibrium in Section 3.2, examine some property of the equilibrium outcomes in Section 3.3, discuss two special market structures (i.e., oligopoly and monopoly) and compare their equilibrium outcomes in Sections 3.4 and 3.5, respectively.

3.1. Model formulation

We consider a *partially concentrated* aftermarket for a durable product (that is sold with a manufacturer's base warranty of length W_b), where n ($n \geq 2$) differentiated EW contracts, denoted by $\mathcal{N} := \{1, 2, \ldots, n\}$, are offered by k ($1 \leq k \leq n$) firms. The firms might include the product's manufacturer, retailers, and other specialized EW providers. Each of the first $k-1$ firms sells a single contract, while the remaining one offers $n-k+1$ contracts that essentially form an EW menu. For simplicity, we refer to the first $k-1$ firms as single-contract firms and the last one as multi-contract firm. It should be noted that partial concentration contains monopoly and oligopoly as two special structures. Specifically, $k = 1$ implies that a monopoly firm offers all the n contracts, while $k = n$ means that each contract is offered by a separate, independent firm.

In the EW pricing game, we only consider the consumers who have bought, or at least have decided to buy, the durable product—that is, *product buyers*. Due to the ancillary nature of EWs, it is indeed meaningless for consumers to purchase EWs without buying the durable product. This consideration enables us to focus exclusively on price competition in EWs. In a static setting, the total number of product units sold—that is, the product's sales volume (denoted by D)—is exactly the market potential for EWs (Wang et al., 2020). As the market potential D is fixed and exogenous for our problem,[4] the price competition game is tackled on a *per-unit* basis. We adopt the MNL model to describe how the product buyers select from the substitutable EW contracts. The MNL model stipulates that each product buyer purchases at most one of the n EW contracts, by trading off price differentials to be decided by the competing firms and variations in EW attributes that are exogenous and already specified by the firms (McFadden, 1974, 2001).

Suppose that each EW contract $i \in \mathcal{N}$ comes with price p_i and attributes $\boldsymbol{X}_i$, where $\boldsymbol{X}_i$ is a vector of observable non-price EW attributes. Typical examples of such attributes are protection length, coverage (components and services covered), deductible, the number and degree of scheduled maintenance, among others. If product buyers purchase EW contract $i \in \mathcal{N}$, then the utility they would gain can be expressed as

$$U_i = u_i + \epsilon_i = v_i - p_i + \epsilon_i, \tag{1}$$

which includes a deterministic component u_i and a random component ϵ_i that captures random utility shocks affected by unobservable characteristics. In this expression, $v_i = v_i(\boldsymbol{X}_i)$ represents the EW contract's gross valuation (also called reserve price), which stems from the compensation for product failures. Since our focus is on price competition, we treat v_i as an exogenous variable, provided that each firm i predetermines attributes $\boldsymbol{X}_i$. Nevertheless, a feasible formulation of v_i is presented in Appendix A for completeness.

Without loss of generality, we normalize the deterministic component u_0 for the no-purchase or outside option to 0; thus, we have $U_0 = \epsilon_0$. For notational convenience, let $\mathcal{N}_+ = \mathcal{N} \cup \{0\}$ be the set of all EW contracts plus the outside option. We further assume that ϵ_i's are independent and identically distributed Gumbel random variables with distribution function $G(x) = \exp(-\exp(-(x/\mu + \chi)))$, where $\chi \approx 0.5772$ is Euler's constant and $\mu > 0$ is a scale parameter measuring the degree of heterogeneities in consumer tastes (Anderson et al., 1992). As $\mathbb{E}[\epsilon_i] = 0$, the deterministic component u_i is actually the average utility of contract $i \in \mathcal{N}$.

The MNL model stipulates that product buyers act as utility-maximizers; that is, they would choose the EW contracts that maximize their *ex post* utilities after the utility uncertainties regarding all contracts plus the outside one are fully resolved. Through random

[4] In Section 4, we shall model the product's sales volume D as a function of product price, so it will be endogenously determined by the manufacturer's product pricing decision.

utility maximization (see, e.g., Anderson et al., 1992), the probability that product buyers would purchase contract $i \in \mathcal{N}$ can be derived as

$$q_i(\boldsymbol{p}) = \Pr\left\{U_i = \max_{j\in\mathcal{N}_+} U_j\right\} = \frac{a_i}{1+\sum_{j\in\mathcal{N}} a_j}, \tag{2}$$

and the no-purchase probability is thus

$$q_0(\boldsymbol{p}) = 1 - \sum_{i\in\mathcal{N}} q_i(\boldsymbol{p}) = \frac{1}{1+\sum_{j\in\mathcal{N}} a_j}, \tag{3}$$

where $a_i = \exp\{(v_i - p_i)/\mu\}$ is defined as the *attractiveness* of each contract $i \in \mathcal{N}$. To simplify notation, we suppress the argument of function q_i, $\forall i \in \mathcal{N}_+$.

It is noteworthy that the purchase probability q_i for each contract $i \in \mathcal{N}$ is strictly decreasing in its own price p_i and increasing in the price p_j of any other contract $j \in \mathcal{N} \setminus \{i\}$, implying that the EWs are *gross substitutes*. This is because $\partial q_i/\partial p_i = -\frac{1}{\mu} q_i(1-q_i) < 0$, $i \in \mathcal{N}$, and $\partial q_i/\partial p_j = \frac{1}{\mu} q_i q_j > 0$, $j \in \mathcal{N} \setminus \{i\}$.

Moreover, the total market size captured by EW contract i is $D \cdot q_i$, while the associated market share is $s_i = q_i/Q$, where $Q = \sum_{j\in\mathcal{N}} q_j$ is the total purchase probability for all the n contracts. It should be noted here that market size represents the level at which a firm occupies the entire aftermarket, whereas market share reflects the relative market power across firms (Xie et al., 2021). An increase in market share does not necessarily correspond to a growth in market size, and vice versa. In addition to market size/share, another key metric is consumer surplus. Under the MNL model, the consumer surplus is defined as the expectation of the maximum utility among all available contracts plus the outside option (see, e.g., Anderson et al., 1992):

$$\begin{aligned} CS &= \mathbb{E}\Big[\max\{v_i - p_i + \epsilon_i : i \in \mathcal{N}_+\}\Big] \\ &= \mu \ln\left(1 + \sum_{i\in\mathcal{N}} a_i\right) = \mu \ln\left(\frac{1}{q_0}\right). \end{aligned} \tag{4}$$

Furthermore, providing EW contracts is by no means free from the firms' perspective, since it is usually costly to service warranty claims made by consumers (Liu & Wang, 2023; Luo & Wu, 2019). We denote by $c_i(\boldsymbol{X}_i)$ the expected marginal cost of honoring EW contract i. This cost is closely related to the product's reliability characteristic and the EW attributes (e.g., repair policy, protection length, and coverage, among others). In this work, as our focus is on price competition and EW attributes $\boldsymbol{X}_i$ are assumed to be predetermined, we do not specify any form for c_i and simply consider it as a fixed and exogenous parameter.

3.2. Pricing game and the equilibrium

We consider a full-information non-cooperative game in which the k firms know all model parameters and have perfect information on the EW prices specified by their competitors. Moreover, all firms act simultaneously. For ease of presentation, let $\mathcal{N}_{k^-} := \{1, 2, \dots, k-1\}$ denote the set of EW contracts offered by the single-contract firms and $\mathcal{N}_k := \{k, k+1, \dots, n\}$ represent the set of those offered by the multi-contract firm. Further let $\boldsymbol{p}_{-i} := (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n)$ denote the EW prices set by the competitors of each single-contract firm $i \in \mathcal{N}_{k^-}$, while $\boldsymbol{p}_k := (p_k, \dots, p_n)$ and $\boldsymbol{p}_{-k} := (p_1, \dots, p_{k-1})$ represent the EW prices set by firm k and its competitors, respectively.

The optimization problem facing each single-contract firm $i \in \mathcal{N}_{k^-}$ is to determine an optimal EW price to maximize the expected *per-unit* EW profit (i.e., the expected EW profit per unit of product sold), given competitor prices $\boldsymbol{p}_{-i}$. Mathematically, the problem for each single-contract firm $i \in \mathcal{N}_{k^-}$ is expressed as

$$\max_{p_i \ge c_i} \ \pi_i(p_i; \boldsymbol{p}_{-i}) = (p_i - c_i) q_i. \tag{5}$$

Similarly, the multi-contract firm's problem is to seek optimal EW prices $\boldsymbol{p}_k$, given $\boldsymbol{p}_{-k}$, to maximize the expected per-unit profit as well:

$$\max_{\boldsymbol{p}_k \ge \boldsymbol{c}_k} \ \pi_k(\boldsymbol{p}_k; \boldsymbol{p}_{-k}) = \sum_{i\in\mathcal{N}_k} (p_i - c_i) q_i, \tag{6}$$

where $\boldsymbol{c}_k := (c_k, c_{k+1}, \dots, c_n)$ represent the EW servicing costs for firm k. Let $m_i = p_i - c_i$ be the markup for each contract $i \in \mathcal{N}$.

The logit-type demand model in (2) ensures that the profit function for each firm is strictly quasi-concave in the associated EW price(s), leading to a unique best response given the competitors' pricing decisions. The following theorem shows that there exists a unique Nash equilibrium and the price equilibria coincide with the unique solutions to the system of first-order-condition equations.

Theorem 1. *There is a unique equilibrium price for each single-contract firm:*

$$p_i^* = c_i + \frac{\mu}{1 - q_i^*}, \ i \in \mathcal{N}_{k^-}, \tag{7}$$

with the equilibrium purchase probability and expected per-unit profit being $q_i^* = \pi_i^*/(\mu + \pi_i^*)$ *and* $\pi_i^* = p_i^* - c_i - \mu$*, respectively.*

Moreover, there are also unique equilibrium prices for the multi-contract firm k*, given by*

$$p_i^* = c_i + \frac{\mu}{1 - \sum_{j\in\mathcal{N}_k} q_j^*}, \ i \in \mathcal{N}_k, \tag{8}$$

with the equilibrium purchase probability and expected per-unit profit given by $\sum_{j\in\mathcal{N}_k} q_j = \pi_k^*/(\mu + \pi_k^*)$ *and* $\pi_k^* = p_i^* - c_i - \mu$, $i \in \mathcal{N}_k$*, respectively.*

We can observe that the equilibrium price for each contract $i \in \mathcal{N}$ has a *cost-plus-markup* structure. Specifically, the markups for EW contracts offered by the single-contract firms are different yet dependent on their own choice probabilities (i.e., $m_i^* = \mu/(1-q_i^*)$, $\forall i \in \mathcal{N}_{k^-}$), whereas the contracts offered by the multi-contract firm have the same markup, which is dependent on the choice probabilities of all contracts managed by that firm (i.e., $m_i^* = \mu/(1-\sum_{j\in\mathcal{N}_k} q_j^*)$, $\forall i \in \mathcal{N}_k$). The "equal-markup" policy has been found in many prior studies on monopolistic multi-product pricing (Li & Huh, 2011; Wang, 2012; Wang et al., 2022), including studies on EW-menu pricing (Wang, 2023; Wang et al., 2020). This is because the price sensitivities across all alternatives are identical in the standard MNL model (Li & Huh, 2011). In this sense, the $n-k+1$ contracts offered by the multi-contract firm compose a concentrated portion of the aftermarket, as if acting in a monopoly.

It is worth mentioning that Eq. (7) can be rewritten as $1/m_i^* = -(\partial q_i/\partial p_i)/q_i^*$, $i \in \mathcal{N}_{k^-}$. In particular, $-\partial q_i/\partial p_i$ is the measure of marginal consumers who are indifferent between contract i and the best alternative (i.e., $v_i - p_i + \epsilon_i = \max\{\epsilon_0, \max_{j\neq i}\{v_j - p_j + \epsilon_j\}\}$). This implies that the equilibrium markup m_i^* is inversely proportional to the proportion of marginal consumers among those who purchase contract i, $\forall i \in \mathcal{N}_{k^-}$. This is a standard optimal pricing formula in the economics literature (Choi et al., 2018). Similar phenomenon can be found for the contracts offered by firm k, noting that (8) can be rewritten as $1/m_i^* = -(\partial \sum_{j\in\mathcal{N}_k} q_j/\partial p_i)/q_i^*$, $i \in \mathcal{N}_k$.

Furthermore, we note that Eq. (7) represents an implicit function of the equilibrium price p_i^* itself, because q_i^* on the right-hand side is a function of p_i^*. Moreover, Eq. (8) is also an implicit function of p_i^*, $\forall i \in \mathcal{N}_k$, because all of the choice probabilities $q_k, q_{k+1}, \dots, q_n$ are functions of p_i^*. This raises the problem of how to compute the equilibrium prices, which is under-explored in the current literature. We now provide an efficient computation procedure, following the spirit of Li and Huh (2011)'s method.

Define $A_i := \exp\{\frac{v_i - c_i}{\mu}\}$, $\forall i \in \mathcal{N}$, which represents the cost-adjusted attractiveness and can be referred to as *effective attractiveness* (Li & Webster, 2024). Notice that according to (2) and (3), for any $i \in \mathcal{N}_{k^-}$,

$$\frac{q_i}{q_0} = a_i = \exp\left\{\frac{v_i - p_i}{\mu}\right\} = A_i \cdot \exp\left\{-\frac{1}{1-q_i}\right\}, \tag{9}$$

where the last equality follows from (7). This equation is equivalent to

$$q_0 = f_i(q_i), \tag{10}$$

where $f_i(q_i) := \frac{q_i}{A_i} \exp\{\frac{1}{1-q_i}\}$, $i \in \mathcal{N}_{k^-}$. We can see that $f_i(q_i)$ is increasing in q_i from $f_i(0^+) = 0$ to $f_i(1^-) = \infty$. As a result, for any given $q_0 \in (0,1)$, there must exist a unique solution $q_i \in (0,1)$ that satisfies (10). Let $\Phi_i(q_0)$ denote this solution for any fixed q_0; that is, $q_i = \Phi_i(q_0) = f_i^{-1}(q_0)$, where $f_i^{-1}(\cdot)$ is the inverse function of $f_i(\cdot)$. Since $f_i(q_i)$ is increasing in q_i, we know that $\Phi_i(q_0)$ is increasing in q_0.

Moreover, for any $i \in \mathcal{N}_k$, substituting (8) into $q_i/q_0 = a_i$ yields

$$\frac{q_i}{q_0} = A_i \cdot \exp\left\{-\frac{1}{1-\sum_{j\in\mathcal{N}_k} q_j}\right\}. \tag{11}$$

Define $\check{f}_i(q_i) := \frac{q_i}{A_i}\exp\{\frac{1}{1-\sum_{j\in\mathcal{N}_k} q_j}\}$, $i \in \mathcal{N}_k$. It is straightforward that for any $q_0 \in (0,1)$ and $\sum_{j\in\mathcal{N}_k\setminus\{i\}} q_j \in (0, 1-q_0)$, there must exist a unique solution $q_i \in (0, 1-\sum_{j\in\mathcal{N}_k\setminus\{i\}} q_j)$ to $q_0 = \check{f}_i(q_i)$, $i \in \mathcal{N}_k$. For any fixed q_0, we denote the unique solution by $\check{\Phi}_i(q_0)$, which is increasing in q_0.

To guarantee the existence of an equilibrium, we should have

$$\sum_{i\in\mathcal{N}_{k^-}} \Phi_i(q_0) + \sum_{i\in\mathcal{N}_k} \check{\Phi}_i(q_0) + q_0 = 1. \tag{12}$$

Notice that $\lim_{q_0\to 0^+} \sum_{i\in\mathcal{N}_{k^-}} \Phi_i(q_0) + \sum_{i\in\mathcal{N}_k} \check{\Phi}_i(q_0) + q_0 = 0$ and $\lim_{q_0\to 1^-} \sum_{i\in\mathcal{N}_{k^-}} \Phi_i(q_0) + \sum_{i\in\mathcal{N}_k} \check{\Phi}_i(q_0) + q_0 \geq 1$. Therefore, there must exist a unique solution $q_0^* \in (0,1)$ that satisfies (12).

Based on the arguments above, we design a simple bisection-type algorithm (i.e., Algorithm 1 in Appendix B), which is employed in the following toy example to illustrate the equilibrium results in Theorem 1. We note that the parameter setting of this example will be revisited to illustrate the analytical results in the subsequent sections.

Example 1. Consider a hypothetical electronic appliance that is sold with a base warranty. The aftermarket for this appliance is partially concentrated, with four firms offering five EWs (i.e., $k = 4$, $n = 5$). Each firm $i \in \{1,2,3\}$ offers a single EW contract at price p_i, while firm 4 offers two EW contracts at prices p_4 and p_5, respectively. The valuations $\{v_i\}$, marginal costs $\{c_i\}$, and effective attractiveness $\{A_i\}$ for all the five contracts are presented in columns 2–4 of Table 1. We set the scale parameter of MNL to $\mu = 15$. The equilibrium outcomes, including EW price p_i^*, markup m_i^*, purchase probability q_i^*, market share s_i^*, and expected per-unit profit π_i^*, for each contract i are summarized in Table 1. At equilibrium, the total purchase probability is $Q^* = \sum_{i=1}^5 q_i^* = 93.61\%$ (i.e., 93.61% of product buyers would like to purchase EWs); the resulting consumer surplus is $CS^* = 41.26$. We can see that the equilibrium markups for contracts 4 and 5 offered by the same firm are identical, which is consistent with the analytical result in Theorem 1.

3.3. An equilibrium property

We now discuss an equilibrium property with respect to an important model parameter—the effective attractiveness (A_i).

Corollary 1. *(i) For any single-contract firm $i \in \mathcal{N}_{k^-}$, at equilibrium, a higher effective attractiveness A_i leads to a higher attractiveness a_i^*, a larger markup m_i^*, a higher purchase probability q_i^*, a larger market share s_i^*, and a higher expected per-unit profit π_i^*, and vice versa. (ii) The same claim also applies to each contract $i \in \mathcal{N}_k$ offered by the multi-contract firm.*

Corollary 1(i)—already revealed by Anderson and De Palma (2001, Proposition 1)—indicates that, at equilibrium, the competitiveness of each single-contract firm's EW can be defined by either of the following metrics: effective attractiveness, attractiveness, markup, purchase probability, market share, and expected per-unit profit. This result also holds for the contracts offered by the multi-contract firm, as informed by Corollary 1(ii). The results are confirmed by the equilibrium outcomes presented in Table 1. It should be emphasized that the comparative statics in Corollary 1 do not hold globally for all the n contracts, yet only hold locally for the contracts offered by the single-contract firms or the multi-contract firm, respectively (see the proof of Corollary 1 for explanation).

Table 1
Equilibrium outcomes in the partially concentrated aftermarket.

i	v_i	c_i	A_i	p_i^*	m_i^*	a_i^*	q_i^* (%)	s_i^* (%)	π_i^*
1	97.29	67.20	7.43	84.79	17.59	2.30	14.71	15.71	2.59
2	93.55	55.16	12.93	74.56	19.40	3.55	22.67	24.21	4.40
3	89.01	47.98	15.41	68.16	20.18	4.02	25.66	27.41	5.18
4	83.99	45.68	12.86	67.29	21.61	3.04	19.46	20.79	4.20
5	78.25	48.33	7.35	69.94	21.61	1.74	11.12	11.88	2.40

Among the six metrics, only the effective attractiveness is primitive, as it is exogenous to the pricing decisions. In other words, firms should strive to provide high effective-attractiveness EW contracts. Notice—from the definition $A_i := \exp\{\frac{v_i-c_i}{\mu}\}$—that an EW with more attractive attributes may not necessarily have a higher effective attractiveness. This is because, while the valuation v_i would be higher, the associated marginal cost c_i becomes higher as well. Therefore, the inherent competitiveness of each EW contract hinges on whether it can offer high valuation to consumers at low marginal cost to the provider.

3.4. Special market structures

We now discuss two special cases of the base model regarding the number of firms, k, in the aftermarket. In particular, $k = 1$ corresponds to a *monopoly* in which a single firm offers all the n contracts, while $k = n$ corresponds to an *oligopoly* in which each contract is offered by a separate, independent firm. Before delving into detailed discussions, we emphasize that the results presented in this subsection either are already established in the literature or can be readily derived from existing findings; however, they all follow directly as corollaries of our general result in Theorem 1.

Corollary 2 (*Oligopoly*). *When $k = n$, the equilibrium price of each contract $i \in \mathcal{N}$ is given by*

$$p_i^\dagger = c_i + \frac{\mu}{1-q_i^\dagger}, \tag{13}$$

where $q_i^\dagger = \Phi_i(q_0^\dagger) = f_i^{-1}(q_0^\dagger)$ for each $i \in \mathcal{N}$ and $q_0^\dagger$ is the unique solution to $\sum_{i\in\mathcal{N}} \Phi_i(q_0) + q_0 = 1$. Moreover, the equilibrium purchase probability and expected per-unit profit for each contract are $q_i^\dagger = \pi_i^\dagger/(\mu + \pi_i^\dagger)$ and $\pi_i^\dagger = p_i^\dagger - c_i - \mu$, respectively.

Though the equilibrium price in (7) and (13) has been derived in early oligopolistic price competition studies under MNL demand (see, e.g., Anderson & De Palma, 2001; Anderson et al., 1992), our work generalizes this result to a partial concentrated aftermarket consisting of a multi-contract firm and multiple single-contract firms. This partial concentration scenario is more general and is also aligned with the current situation in the EW market.

It should be noted that the equilibrium price in Eq. (13) can be expressed in an alternative form involving the Lambert W function (Corless et al., 1996); that is,

$$p_i^\dagger = c_i + \mu\left[W\left(\frac{A_i e^{-1}}{1+\sum_{j\neq i} a_j}\right) + 1\right], \tag{14}$$

where $W(z)$ is the solution x to $xe^x = z$ for any $z \geq 0$. The derivation of (14) can be found in Online Supplement. Similar results have been derived by Li and Huh (2011, Theorem 4) and Loots and den Boer (2023, Eq. (21)). The former research addresses price competition in an oligopoly under the nested logit model (with MNL as a special case), while the latter focuses on collusion and competition in a pricing duopoly with zero marginal costs under the MNL model.

We then examine the monopoly setting (i.e., $k = 1$) in which the monopolistic firm's problem becomes $\max_{p\geq c} \sum_{i\in\mathcal{N}} (p_i - c_i)q_i$, which has been studied by Wang et al. (2020) in the context of EW-menu design and pricing. The problem is also referred to as a joint-profit maximization problem by Loots and den Boer (2023) from a collusion perspective.

Corollary 3 (*Monopoly*). *Let $\tilde{f}(q_i) := \frac{q_i}{A_i}\exp\{\frac{1}{1-\sum_{j\in\mathcal{N}} q_j}\}$. When $k = 1$, the equilibrium price of each contract $i \in \mathcal{N}$ is given by*

$$p_i^{\ddagger} = c_i + \frac{\mu}{1 - \sum_{j\in\mathcal{N}} q_j^{\ddagger}}, \tag{15}$$

where $q_i^{\ddagger} = \tilde{\Phi}_i(q_0^{\ddagger}) = \tilde{f}^{-1}(q_0^{\ddagger})$ for each $i \in \mathcal{N}$ and $q_0^{\ddagger}$ is the unique solution to $\sum_{i\in\mathcal{N}} \tilde{\Phi}_i(q_0) + q_0 = 1$. Moreover, the resultant total purchase probability is $Q^{\ddagger} = \pi^{\ddagger}/(\pi^{\ddagger} + \mu)$ and the consumer surplus is $CS^{\ddagger} = \mu \ln(1 + \pi^{\ddagger}/\mu)$.

We can see that similar to the concentrated portion (i.e., EW contracts offered by firm k) in Theorem 1, the equilibrium pricing policy in the monopoly setting is also a *cost-plus-markup* policy, with the same markup for all contracts (i.e., $m_i^{\ddagger} = \mu/(1 - \sum_{j\in\mathcal{N}} q_j^{\ddagger})$, $\forall i \in \mathcal{N}$). This equal-markup policy is again due to the identical price sensitivities across all alternatives in the standard MNL model.

We note that the equilibrium price $p_i^{\ddagger}$ in (15) can be expressed in two alternative forms. The first is based on the ancillary-variable transformation method proposed by Wang (2012) and adopted by Wang et al. (2020). Specifically, $p_i^{\ddagger} = c_i + \pi^{\ddagger} + \mu$, where $\pi^{\ddagger}$—the maximum expected EW profit per unit sold—is the unique solution to

$$\pi = \mu \sum_{i\in\mathcal{N}} \exp\left\{\frac{v_i - c_i - \pi - \mu}{\mu}\right\}. \tag{16}$$

This form has been derived by Wang et al. (2020, Theorem 1), thus the proof is omitted. A similar result can be found in the work of Loots and den Boer (2023, Eq. (5)), who focus on a pricing duopoly with zero marginal costs and asymmetric price sensitivities. The second form is based on the Lambert W function:

$$p_i^{\ddagger} = c_i + \mu\left[W\left(e^{-1}\sum_{i\in\mathcal{N}} A_i\right) + 1\right]. \tag{17}$$

A similar result has been derived by Li and Huh (2011, Corollary 1). We provide the derivation in Online Supplement for self-containment.

3.5. Comparison of equilibrium outcomes

Examining Theorem 1 and Corollaries 2–3 reveals that the partially concentrated aftermarket behaves like a combined oligopoly–monopoly in the sense that the single-contract firms act as in an oligopoly (i.e., they set different markups for their EWs), while the multi-contract firm acts as in a monopoly (i.e., it adopts an equal-markup pricing policy for its EWs). We now compare the equilibrium outcomes under the partial concentration, oligopoly, and monopoly settings to investigate the impact of market-concentration degree on the equilibrium outcomes.

Proposition 1. *Comparing the partial concentration, oligopoly, and monopoly settings yields the following properties:*

(i) $p_i^{\dagger} \le p_i^* \le p_i^{\ddagger}$, $\forall i \in \mathcal{N}$;
(ii) $Q^{\dagger} \ge Q^* \ge Q^{\ddagger}$ *and* $CS^{\dagger} \ge CS^* \ge CS^{\ddagger}$.

Further comparing the partial concentration and oligopoly settings, in terms of purchase probabilities, market shares, and expected per-unit profits, leads to additional properties:

(iii) $q_i^{\dagger} \le q_i^*$ *and* $s_i^{\dagger} \le s_i^*$ *for* $i \in \mathcal{N}_{k-}$, *and* $\sum_{i\in\mathcal{N}_k} q_i^{\dagger} \ge \sum_{i\in\mathcal{N}_k} q_i^*$;
(iv) $\pi_i^{\dagger} \le \pi_i^*$ *for* $i \in \mathcal{N}_{k-}$.

Proposition 1(i) shows that the equilibrium price of each EW contract in an oligopoly is lower than that in a partially concentrated market, which is further lower than that in a monopoly; however, the situation is reversed for the total purchase probability and consumer surplus (see part (ii)). This is because when the EW market becomes more concentrated (i.e., the n contracts are offered by fewer firms), the competition intensity is relieved. As a consequence, the firms are able to charge a higher price for each contract, which in turn harms consumer welfare. Moreover, the total purchase probability also shrinks due to the increase in EW prices.

We further compare the oligopoly and partially concentrated settings in a more detailed manner. Parts (iii) and (iv) of Proposition 1 show that compared with the oligopoly, in the partially concentrated setting the combined purchase probabilities for contracts $k, k+1, \dots, n$ become smaller, while each of the other contracts has a larger purchase probability, market share, and expected per-unit profit. This is again due to the increase in EW prices under partial market concentration, which leads to a lower competition intensity. Though the total purchase probability shrinks because of the increased EW prices, each contract offered by the single-contract firms gains a larger purchase probability, market share, and expected per-unit profit. However, comparing individual profits for contracts $k, k+1, \dots, n$ under partial concentration, oligopoly, and monopoly is non-trivial. Nevertheless, Loots and den Boer (2023) numerically show that in a pricing duopoly under MNL demand, joint-profit maximization (analogous to the monopoly setting in our work) is not always mutually profitable to both firms compared to the Nash equilibrium.

In general, the result in Proposition 1 is consistent with our intuition: market concentration softens competition, which, in turn, would harm consumers. This result supports the call for more competition in the electrical-goods EW markets by the UK Competition Commission (2003). We present the following example to demonstrate the results in Corollaries 2–3 and Proposition 1.

Example 2. We adopt the same parameter setting as in Example 1. The equilibrium outcomes for the oligopoly and monopoly settings are presented in Table 2. In particular, the equilibrium total purchase probability and consumer surplus are $Q^{\dagger} = 94.13\%$ and $CS^{\dagger} = 42.50$ in the oligopoly and $Q^{\ddagger} = 68.99\%$ and $CS^{\ddagger} = 17.57$ in the monopoly.

We can see that the numerical results in Tables 1 and 2 confirm the comparative findings in Proposition 1. Fig. 1 also visualizes some key metrics under the three market structures. Interestingly, examining Tables 1 and 2 shows that compared with the oligopoly, partial concentration leads to only a slight increase in the prices of contracts 1–3, but a much larger price growth for contracts 4 and 5. This explains why contracts 1–3 can have larger purchase probabilities (market shares) and extract more profits while lifting their prices. Moreover, even though the combined purchase probabilities of contracts 4 and 5 become smaller in the partially concentrated setting, the associated expected per-unit profits indeed increase, thanks to the significantly increased prices.

Furthermore, compared with the oligopoly, the monopoly leads to significantly higher price, markup, and expected per-unit profit for each EW contract (notably, the expected per-unit profit is almost doubled); by contrast, it results in significantly lower total purchase probability and consumer surplus. This attains the so-called "collusion" in the context of Loots and den Boer (2023). In addition, the purchase probability of each contract under the monopoly is smaller than its counterpart under the oligopoly. However, this is not the case for market shares—some contracts (i.e., 2, 3, and 4) gain larger market shares under the monopoly, while others (i.e., 1 and 5) lose a certain proportion of market share. This is consistent with our previous claim that an increase in market share does not necessarily correspond to a growth in purchase probability, and vice versa.

Furthermore, we compare the purchase probabilities for each individual EW contract under the monopoly and oligopoly in an analytical fashion. To this end, we first present the following insight regarding the impact of competition on the *relative* purchase probability of any pair of EW contracts.

Proposition 2 (*Market-share redistribution*). *If the n contracts are labeled in descending order of A_i (i.e., $A_1 \ge A_2 \ge \cdots \ge A_n > 0$), then for any $j \ge i$,*

$$\frac{q_i^{\dagger}}{q_j^{\dagger}} \le \frac{q_i^{\ddagger}}{q_j^{\ddagger}}.$$

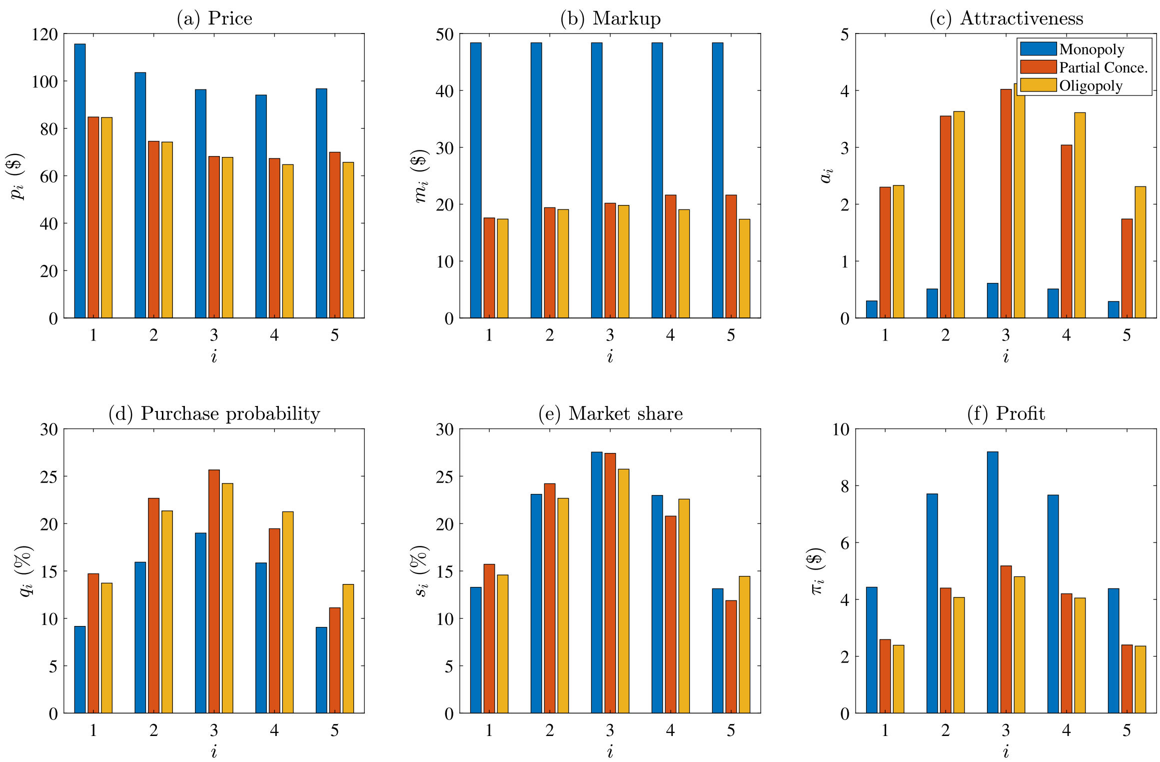

Fig. 1. Equilibrium outcomes in the partially concentrated, monopoly, and oligopoly settings.

Table 2
Equilibrium outcomes in the oligopoly and monopoly settings.

i	$p_i^{\dagger}$	$m_i^{\dagger}$	$a_i^{\dagger}$	$q_i^{\dagger}$ (%)	$s_i^{\dagger}$ (%)	$\pi_i^{\dagger}$
1	84.59	17.39	2.33	13.72	14.58	2.39
2	74.23	19.07	3.63	21.34	22.67	4.07
3	67.78	19.80	4.12	24.23	25.74	4.80
4	64.73	19.05	3.61	21.25	22.58	4.05
5	65.69	17.36	2.31	13.59	14.44	2.36
i	$p_i^{\ddagger}$	$m_i^{\ddagger}$	$a_i^{\ddagger}$	$q_i^{\ddagger}$ (%)	$s_i^{\ddagger}$ (%)	$\pi_i^{\ddagger}$
1	115.58	48.38	0.30	9.16	13.28	4.43
2	103.54	48.38	0.51	15.93	23.09	7.71
3	96.36	48.38	0.61	19.00	27.53	9.19
4	94.06	48.38	0.51	15.85	22.97	7.67
5	96.71	48.38	0.29	9.06	13.13	4.38

Proposition 2 indicates that when the market structure shifts from monopoly to oligopoly, compared to a smaller effective-attractiveness contract, a larger effective-attractiveness contract would sacrifice its purchase probability in a relative manner. We refer to this phenomenon as *market-share redistribution* induced by increased competition. The insight in Proposition 2 is confirmed by the equilibrium outcomes in Table 2. We can see that when the market structure shifts from monopoly to oligopoly, the market share of contract 3 (the one with the largest effective attractiveness) decreases from 27.53% to 25.74%, while that for contract 5 (the one with the smallest effective attractiveness) increases from 13.13% to 14.44%. This indicates that oligopolistic competition entails market-share redistribution in the sense that a proportion of large-effective-attractiveness contracts' market shares is redistributed to those with small effective attractiveness.

The result in Proposition 2 provides a basis for comparing each individual contract's purchase probabilities under the monopoly and oligopoly, which is presented in the following corollary.

Corollary 4. *If the n contracts are labeled in descending order of A_i, then there exists $\tau \in \mathcal{N}$ such that $q_i^{\dagger} \le q_i^{\ddagger}$ for $i < \tau$ and $q_i^{\dagger} \ge q_i^{\ddagger}$ for $i \ge \tau$.*

In the following example, we demonstrate that when the difference between A_i and A_j, $i < j$, becomes extremely large (i.e., $A_i \gg A_j$), the associated purchase probability $q_i^{\dagger}$ in the oligopoly might be lower than its counterpart $q_i^{\ddagger}$ in the monopoly, although competition can raise the total purchase probability (i.e., $Q^{\dagger} \ge Q^{\ddagger}$).

Example 3. We use the same parameter setting as in Example 1, except that the marginal cost of contract 3, c_3, is reduced from 47.98 to 26.98. Accordingly, the associated effective attractiveness increases to $A_3 = 62.51$, which becomes much larger than those of the other contracts. We find from Table 3 that the equilibrium price of contract 3 decreases in both market settings, as the marginal cost is reduced significantly. Nevertheless, at equilibrium the markup, attractiveness, purchase probability, market share, and expected per-unit profit all follow the order of effective attractiveness (Corollary 1). Consistent with Proposition 1, the total purchase probability in the monopoly ($Q^{\ddagger} = 72.65\%$) is smaller than that in the oligopoly ($Q^{\dagger} = 95.88\%$). By contrast, the purchase probability of contract 3 in the former setting ($q_3^{\ddagger} = 44.06\%$) is larger than that in the latter ($q_3^{\dagger} = 43.64\%$). This is because of the market-share redistribution effect induced by competition (Proposition 2).

4. Price competition in product and extended warranties

Since EWs are essentially ancillary after-sales services, it is meaningless for consumers to buy an EW if the main product was not bought. In other words, consumers' purchase decisions on EWs should be conditional on their already purchasing, or at least deciding to purchase, the main product. To accommodate this ancillary nature, we extend the analysis to price competition in the main product and EWs, where the prices of the main product and the optional EWs are characterized by

Table 3
Equilibrium outcomes in the oligopoly and monopoly settings when $c_3 = 26.98$.

i	$p_i^\dagger$	$m_i^\dagger$	$a_i^\dagger$	$q_i^\dagger$ (%)	$s_i^\dagger$ (%)	$\pi_i^\dagger$
1	83.88	16.68	2.45	10.06	10.49	1.68
2	73.05	17.89	3.92	16.14	16.83	2.89
3	53.59	26.61	10.60	43.64	45.52	11.61
4	63.55	17.87	3.91	16.08	16.77	2.87
5	64.99	16.66	2.42	9.96	10.39	1.66
i	$p_i^\ddagger$	$m_i^\ddagger$	$a_i^\ddagger$	$q_i^\ddagger$ (%)	$s_i^\ddagger$ (%)	$\pi_i^\ddagger$
1	122.07	54.87	0.19	5.24	7.21	2.88
2	110.03	54.87	0.33	9.11	12.54	5.00
3	81.85	54.87	1.61	44.06	60.65	24.18
4	100.55	54.87	0.33	9.06	12.47	4.97
5	103.20	54.87	0.19	5.18	7.13	2.84

an equilibrium to a sequential game. It should be noted here that the base-warranty length is not treated as a decision variable. This is in line with the current practice in that the base warranty period is largely constrained by regulation requirements[5] and/or market competition[6] and there is little room for firms to adjust this variable.

We consider the same partially concentrated aftermarket for EWs as in Section 3.1. Without loss of generality, we suppose that the manufacturer—in addition to being the sole provider of the main product—offers multiple EW contracts. That is, the manufacturer serves as firm k. Our findings would remain valid qualitatively, if another firm, other than the manufacturer, serves as firm k. In particular, we consider two sequential product–EW pricing games: one where the manufacturer (as the market leader) sets product and EW prices *sequentially*, and another where these decisions are made *simultaneously* (Heese, 2012; Wang et al., 2024). In the first scenario, the manufacturer first announces the product price, then competes with the other $k-1$ firms on EW prices. In the second scenario, however, the manufacturer first announces the prices of both the main product and the EWs it offers; then, the remaining $k-1$ firms make their EW pricing decisions simultaneously. The two scenarios are quite representative in real-world retailing of durable products and ancillary EWs (Heese, 2012; Wang et al., 2024). For ease of presentation, we refer to the sequential game under the manufacturer's sequential (resp. simultaneous) retail strategy as sequential game I (resp. sequential game II). In both games, the manufacturer acts as the market leader, whereas the single-contract firms are the followers.

Suppose that there is no fixed cost for manufacturing the main product. Let C_m denote the variable manufacturing cost plus the base-warranty servicing cost for each product unit. We adopt the well-known displaced log-linear model by Glickman and Berger (1976) to describe product sales volume $D(P_b, W_b)$ as a function of product price P_b and base-warranty length W_b. That is, $D(P_b, W_b) = \phi_1 P_b^{-\psi_1}(W_b + \phi_2)^{\psi_2}$, where $\phi_1 > 0$ is an amplitude factor and $\phi_2 > 0$ is a constant of time displacement which allows for the possibility of nonzero demand when W_b is zero; in addition, $\psi_1 > 1$ and $0 < \psi_2 < 1$ are the price elasticity and the displaced warranty period elasticity, respectively. This sales volume model has been widely adopted in warranty research (see, e.g., Huang et al., 2007; Li et al., 2023; Xie et al., 2014). Since the base-warranty length W_b is considered exogenous, the sales volume can be simplified to

$$D(P_b) = \phi P_b^{-\psi}, \tag{18}$$

where $\phi = \phi_1(W_b + \phi_2)^{\psi_2}$ and $\psi = \psi_1$. As can be seen, the sales model in (18) has a constant elasticity ψ; that is, $\psi = -P_b D'(P_b)/D(P_b)$.

[5] For instance, Europe Union law stipulates that manufacturers must offer the consumers a minimum 2-year warranty as a protection against faulty products.

[6] The base warranty periods for most smart phones and laptops are 1 year and 2 years, respectively, while those for most automobiles are 3 years or 36 000 miles, whichever occurs first.

4.1. Sequential game I

We start with the first scenario in which the manufacturer only announces product price at the first stage; then, the manufacturer and the other $k-1$ firms compete on EW prices at the second stage. This scenario is quite similar to that studied by Cohen and Whang (1997), yet they consider a simpler aftermarket with a manufacturer and an independent service provider, each offering an after-sales service (i.e., $n = k = 2$). We analyze the sequential pricing game backward. Note that a breve (˘) is used to represent quantities related to the first scenario.

Stage 2: Given the manufacturer's product pricing decision P_b, the market potential for EWs, $D(P_b)$, is fixed. As a result, the EW pricing problems for the manufacturer and the other $k-1$ firms are $\max_{\boldsymbol{p}_k \geq \boldsymbol{c}_k} \breve{\pi}_k(\boldsymbol{p}_k; \boldsymbol{p}_{-k}) = \sum_{i \in \mathcal{N}_k}(p_i - c_i)q_i$ and $\max_{p_i \geq c_i} \breve{\pi}_i(p_i; \boldsymbol{p}_{-i}) = (p_i - c_i)q_i$, $i \in \mathcal{N}_{k-}$, respectively, which are exactly the same as those in (6) and (5). Therefore, the equilibrium prices at Stage 2 are the same as those presented in Theorem 1, which satisfy $\breve{p}_i^* = c_i + \frac{\mu}{1-\breve{q}_i^*}$, $i \in \mathcal{N}_{k-}$, and $\breve{p}_i^* = c_i + \frac{\mu}{1-\sum_{j\in\mathcal{N}_k}\breve{q}_j^*}$, $i \in \mathcal{N}_k$. The equilibrium prices can be computed by Algorithm 1.

Stage 1: The manufacturer determines an optimal product price to maximize the expected total profit from selling the main product and the EWs, given by

$$\breve{\Pi}_k(P_b; \breve{\boldsymbol{p}}_k^*, \breve{\boldsymbol{p}}_{-k}^*) = D(P_b)\left(P_b - C_m + \breve{\pi}_k^*\right), \tag{19}$$

where $P_b - C_m$ is the markup on the main product and $\breve{\pi}_k^* = \sum_{i\in\mathcal{N}_k}(\breve{p}_i^* - c_i)\breve{q}_i^*$ is the equilibrium per-unit profit from the EWs. In this sense, $D(P_b)\cdot\breve{\pi}_k^*$ is the manufacturer's total EW profit for all units sold.

Solving the problem $\max_{P_b \geq C_m} \breve{\Pi}_k(P_b; \breve{\boldsymbol{p}}_k^*, \breve{\boldsymbol{p}}_{-k}^*)$ yields the following result.

Theorem 2. *At equilibrium, the main product should be priced at*

$$\breve{P}_b^* = \frac{\psi}{\psi - 1}(C_m - \breve{\pi}_k^*). \tag{20}$$

The equilibrium product sales volume and expected total profit for the manufacturer can be obtained by substituting (20) into (18) and (19), respectively, which are given by

$$D(\breve{P}_b^*) = \phi\left(\frac{\psi}{\psi-1}\right)^{-\psi}(C_m - \breve{\pi}_k^*)^{-\psi} \tag{21}$$

and

$$\breve{\Pi}_k(\breve{P}_b^*; \breve{\boldsymbol{p}}_k^*, \breve{\boldsymbol{p}}_{-k}^*) = \phi\psi^{-\psi}\left(\frac{\psi}{\psi-1}\right)^{1-\psi}(C_m - \breve{\pi}_k^*)^{1-\psi}. \tag{22}$$

Similarly, the equilibrium total profit for each single-contract firm $i \in \mathcal{N}_{k-}$, with product sales volume endogenously determined by the manufacturer's product pricing decision, can be obtained by

$$\breve{\Pi}_i(\breve{p}_i^*) = D(\breve{P}_b^*)\cdot\breve{\pi}_i^* = D(\breve{P}_b^*)(\breve{p}_i^* - c_i)\breve{q}_i^*, \tag{23}$$

where $D(\breve{P}_b^*)$ is given by (21).

Theorem 2 shows that the equilibrium product price $\breve{P}_b^*$ is negatively related to the equilibrium per-unit EW profit $\breve{\pi}_k^*$. More specifically, when offering EWs in addition to the main product, the manufacturer would reduce the product price.[7] This is to be expected, as a lower product price would attract more consumers to purchase the main product, thereby creating a larger aftermarket for EW sales. Indeed, we can observe that both the sales volume $D(\breve{P}_b^*)$ and the manufacturer's total profit $\breve{\Pi}_k(\breve{P}_b^*; \breve{\boldsymbol{p}}_k^*, \breve{\boldsymbol{p}}_{-k}^*)$ are positively related to $\breve{\pi}_k^*$ (recall that $\psi > 1$). That is, offering EWs can boost product sales and increase the

[7] According to Glickman and Berger (1976), when the manufacturer sells the main product without EWs, the profit-maximization problem becomes $\max_{P_b\geq C_m} \Pi_k(P_b) = D(P_b)(P_b - C_m)$, leading to an optimal product price $P_b^* = \psi C_m/(\psi - 1)$. The resultant product sales volume and total profit are $D(P_b^*) = \phi(\psi C_m)^{-\psi}/(\psi-1)^{-\psi}$ and $\Pi_k(P_b^*) = \phi\psi^{-\psi}(\psi C_m)^{1-\psi}/(\psi-1)^{1-\psi}$, respectively.

manufacturer's total profit. Overall, Theorem 2 indicates that when the product and EW pricing decisions are made in a sequential manner, the manufacturer would choose to seek a higher overall profit at the sacrifice of the main product's markup. This is consistent with the finding by Wang et al. (2024).

In addition, we can observe that EW price competition at the second stage can affect the manufacturer's product pricing decision at the first stage (through the per-unit EW profit $\check{\pi}_k^*$ in (20)). Even though the reverse is not true (i.e., the product pricing decision does not influence EW price competition), the product price $\check{P}_b^*$ indeed has an impact on the total profit of each single-contract firm (through sales volume $D(\check{P}_b^*)$ in (23)). This implies that despite the simple constant-elasticity nature of the product sales model in (18), our sequential product–EW game can capture the impact of EW price competition on the interaction between product and EW pricing decisions.

4.2. Sequential game II

We then proceed to the second scenario in which the manufacturer first announces the product price and EW prices; then, the remaining $k-1$ firms determine their EW prices simultaneously. We use a hat (ˆ) to represent quantities related to the second scenario.

Stage 2: Given the product price P_b and EW prices $\boldsymbol{p}_k$ announced by the manufacturer, the EW pricing problem for each of the single-contract firms is $\max_{p_i \ge c_i} \hat{\pi}_i(p_i; \boldsymbol{p}_{-i}) = (p_i - c_i)q_i,\ i \in \mathcal{N}_{k-}$, which is again identical to that in (5). Hence, according to Theorem 1, the equilibrium EW price for each single-contract firm, at Stage 2, is given by $\hat{p}_i^* = c_i + \frac{\mu}{1-\hat{q}_i^*},\ i \in \mathcal{N}_{k-}$.

Recall that $\Phi_i(q_0)$ is the unique solution to $q_0 = f_i(q_i)$ in (10) for any $q_0 \in (0,1)$. For any $i \in \mathcal{N}_{k-}$, let

$$\hat{a}_i = \exp\left\{\frac{v_i - \hat{p}_i^*}{\mu}\right\} = A_i \exp\left\{-\frac{1}{1-\Phi_i(q_0)}\right\}$$

be the attractiveness under price $\hat{p}_i^*$. Given $\hat{a}_i$ for $i \in \mathcal{N}_{k-}$, we have

$$\hat{q}_0 = \frac{1}{1+\sum_{j\in\mathcal{N}_{k-}} \hat{a}_j + \sum_{j\in\mathcal{N}_k} a_j} \tag{24}$$

and

$$\hat{q}_i = \frac{a_i}{1+\sum_{j\in\mathcal{N}_{k-}} \hat{a}_j + \sum_{j\in\mathcal{N}_k} a_j},\quad i \in \mathcal{N}_k. \tag{25}$$

Note that all of $\hat{q}_0$ and $\hat{q}_i$, $i \in \mathcal{N}_k$, are functions of $\boldsymbol{p}_k$.

Noticing that $\hat{q}_i = \hat{q}_0 a_i$, $i \in \mathcal{N}$, we should have

$$\sum_{j\in\mathcal{N}_{k-}} \Phi_j(\hat{q}_0) + \sum_{j\in\mathcal{N}_k} \hat{q}_0 a_j + \hat{q}_0 = 1. \tag{26}$$

Clearly, given any fixed $\boldsymbol{p}_k$, there must exist a unique solution $\hat{q}_0 \in (0,1)$ that satisfies (26).

Stage 1: The manufacturer determines optimal product price and EW prices to maximize the expected total profit from selling the main product and EWs, given by

$$\begin{aligned}\hat{\Pi}_k(P_b, \boldsymbol{p}_k; \hat{\boldsymbol{p}}_{-k}^*) &= D(P_b)\left[P_b - C_m + \hat{\pi}_k(\boldsymbol{p}_k; \hat{\boldsymbol{p}}_{-k}^*)\right] \\ &= \phi P_b^{-\psi}\left[P_b - C_m + \sum_{i\in\mathcal{N}_k}(p_i - c_i)\hat{q}_i\right],\end{aligned} \tag{27}$$

where $\hat{q}_i$, $i \in \mathcal{N}_k$, is given by (25).

Let $\Theta = -\sum_{j\in\mathcal{N}_{k-}} \frac{[\Phi_j(\hat{q}_0)]^2}{[1-\Phi_j(\hat{q}_0)]^2+\Phi_j(\hat{q}_0)}$, which satisfies $1 \le \frac{1}{1+\Theta} \le \frac{1}{\sum_{i\in\mathcal{N}_k} \hat{q}_i^*}$ (see the proof of Theorem 3). Solving the first-stage problem $\max_{P_b \ge C_m, \boldsymbol{p}_k \ge \boldsymbol{c}_k} \hat{\Pi}_k(P_b; \boldsymbol{p}_k, \hat{\boldsymbol{p}}_{-k}^*)$ yields the following theorem. How to compute the price equilibria is discussed in Appendix C.

Theorem 3. *At equilibrium, the manufacturer's EWs should be priced at*

$$\hat{p}_i^* = c_i + \frac{\mu}{1 - \frac{1}{1+\Theta}\sum_{j\in\mathcal{N}_k} \hat{q}_j^*},\ \forall i \in \mathcal{N}_k, \tag{28}$$

while the main product should be priced at

$$\hat{P}_b^* = \frac{\psi}{\psi - 1}(C_m - \hat{\pi}_k^*). \tag{29}$$

We can see from Theorem 3 that when the EW pricing game is a sequential game (i.e., the manufacturer moves before the single-contract firms), rather than a simultaneous one, offering EWs still incentivizes the manufacturer to lower the product price. By contrast, the equilibrium prices for the manufacturer's EWs in (28) have a different structure from their counterparts in the simultaneous EW pricing game. In addition, the product sales volume, the manufacturer's total profit and each single-contract firm's total profit, at equilibrium, can still be evaluated by Eqs. (21), (22) and (23), respectively, with the quantities $\hat{p}_i^*$, $\hat{q}_i^*$, and $\hat{\pi}_k^*$ being substituted properly.

4.3. Comparison of equilibrium outcomes

We now conduct a comparison between the equilibrium outcomes of the two sequential games, in order to examine the impact of the manufacturer's retail strategy on the equilibrium. Notice first that the equilibrium EW prices under the sequential game I are the same as those under the simultaneous game in the partially concentrated setting (Theorem 1), as the manufacturer and the other single-contract firms compete in the aftermarket, simultaneously.

Proposition 3. *By comparing EW-related equilibrium outcomes for the two sequential games as well as the simultaneous game, we have*

(i) $p_i^* = \check{p}_i^* \le \hat{p}_i^*,\ i \in \mathcal{N};\ \pi_i^* = \check{\pi}_i^* \le \hat{\pi}_i^*,\ \forall i \in \mathcal{N}_{k-}$.
(ii) $q_0^* = \check{q}_0^* \le \hat{q}_0^*;\ q_i^* = \check{q}_i^* \le \hat{q}_i^*$ *and* $s_i^* = \check{s}_i^* \le \hat{s}_i^*,\ \forall i \in \mathcal{N}_{k-};\ \sum_{i\in\mathcal{N}_k} q_i^* = \sum_{i\in\mathcal{N}_k} \check{q}_i^* \ge \sum_{i\in\mathcal{N}_k} \hat{q}_i^*$ *and* $\sum_{i\in\mathcal{N}_k} s_i^* = \sum_{i\in\mathcal{N}_k} \check{s}_i^* \ge \sum_{i\in\mathcal{N}_k} \hat{s}_i^*$.
(iii) $Q^* = \check{Q}^* \ge \hat{Q}^*,\ CS^* = \check{CS}^* \ge \hat{CS}^*$.

Proposition 3 indicates that compared with the sequential game I, at equilibrium the sequential game II leads to higher EW prices for all firms and higher per-unit EW profits for the single-contract firms; a smaller purchase probability (market share) for the manufacturer and a larger purchase probability (market share) for each single-contract firm; and a smaller total purchase probability for EWs and a lower consumer surplus. In the sequential game II, the manufacturer, as the market leader, has a first-mover advantage, which is absent in the sequential game I. It grants the manufacturer an advantage to charge a higher EW price. The announcement of a higher EW price (i.e., a price concession) by the manufacturer helps to soften price competition in the aftermarket, which enables the single-contract firms to increase their EW prices and extract more per-unit profits. On the other hand, the single-contract firms, as the second mover, have an advantage to wisely set their EW prices in such a way that their EW purchase probabilities (market shares) and per-unit EW profits can be even larger than those in the sequential game I. Nevertheless, the total purchase probability for all EWs becomes smaller due to the increase in EW prices, and thus fewer consumers would buy EWs in the sequential game II, leading to impaired consumer surplus.

Unfortunately, comparing the manufacturer's per-unit EW profits under the two game scenarios (i.e., $\check{\pi}_k^*$ and $\hat{\pi}_k^*$) is non-trivial, although conventional wisdom indicates that compared with the simultaneous game, the existence of a market leader leads to Pareto improving market outcomes for all firms. Nevertheless, what we can speculate is that if the sequential game II results in a higher per-unit EW profit for the manufacturer (i.e., $\pi_k^* = \check{\pi}_k^* \le \hat{\pi}_k^*$), then it would lead to a lower product price, a larger product sales volume, higher total profits for the manufacturer and each single-contract firm, and vice versa.

We use the following example to demonstrate the results in Theorems 2–3 and Proposition 3.

Example 4. We adopt the same parameter setting as in Example 1 and suppose that firm 4 is the manufacturer, the sole product provider.

In addition, we arbitrarily set the following parameter values for illustrative purposes: $\phi = 10^{14}$, $\psi = 2.5$, and $C_m = 500$. The equilibrium outcomes related to EWs under the sequential game II are summarized in Table 4. Those under the sequential game I are identical to the results under the simultaneous game (reported in Table 1). The comparison between Tables 1 and 4 reveals that when the EW pricing game moves from simultaneous (Sequential game I) to sequential (Sequential game II), the EW prices of all firms become higher, although the increase in the manufacturer's EW prices is of a larger extent. By contrast, though all firms' per-unit EW profits become higher as well, the profit growth of the manufacturer is relatively insignificant (from $\breve{\pi}_4^* = 6.608$ to $\hat{\pi}_4^* = 6.627$). Moreover, the manufacturer's EW purchase probabilities (market shares) become smaller, whereas those of the single-contract firms become larger. The total EW purchase probability is reduced from $\breve{Q}^* = 93.61\%$ to $\hat{Q}^* = 93.39\%$, and the consumer surplus is reduced from $\breve{CS}^* = 41.26$ to $\hat{CS}^* = 40.73$.

In addition, we present in Table 5 the equilibrium outcomes related to the main product. As can be seen, the product price under game II is lower than that under game I, leading to a larger sales volume, because the associated EW price is higher than its counterpart. Thanks to the increased EW prices and the expanded aftermarket, the manufacturer's total profit $\hat{\Pi}_k$ is higher than its counterpart $\breve{\Pi}_k$. We also report the EW market size captured by each firm in rows 5–8, along with the associated overall EW profit in rows 9–12 (note that the metrics for the manufacturer are the combined ones for contracts 4 and 5). We can observe that the manufacturer's (resp. each single-contract firm's) EW market size under game II is smaller (resp. larger) than that under game I. On the other hand, both the manufacturer's and single-contract firm's overall EW profits under game II are higher than those under game I. These results are consistent with our speculation mentioned earlier. Overall, we can see that all the concerned metrics for the manufacturer under the two game scenarios are quite close. This observation remains valid if the manufacturer offers another two contracts (say, contracts 3 and 4); the results are omitted for brevity. It implies that the first-mover advantage of the manufacturer as a market leader is insignificant in our context.

Table 4
EW related equilibrium outcomes for the sequential game II.

i	$\hat{p}_i^*$	$\hat{m}_i^*$	$\hat{a}_i^*$	$\hat{q}_i^*$ (%)	$\hat{s}_i^*$ (%)	$\hat{\pi}_i^*$
1	84.88	17.68	2.29	15.14	16.21	2.68
2	74.70	19.54	3.51	23.24	24.88	4.54
3	68.33	20.35	3.97	26.27	28.13	5.34
4	68.74	23.06	2.76	18.29	19.58	4.22
5	71.39	23.06	1.58	10.45	11.19	2.41

Table 5
Product related equilibrium outcomes for the sequential games.

Metrics		Game I	Game II
P_b^*		822.3204	822.2875
D^* ($\times 10^7$)		5.1570	5.1575
Π_k^* ($\times 10^9$)		1.6963	1.6964
$D^* q_i^*$ ($\times 10^6$)	$i = 1$	0.7583	0.7807
	$i = 2$	1.1690	1.1987
	$i = 3$	1.3232	1.3551
	$i = 4$	1.5771	1.4823
$D^* \pi_i^*$ ($\times 10^7$)	$i = 1$	1.3336	1.3799
	$i = 2$	2.2674	2.3425
	$i = 3$	2.6699	2.7570
	$i = 4$	3.4077	3.4182

5. Conclusion

In this paper, we present a game-theoretical model to analyze price competition in an EW market for durable products. The aftermarket is of a partially concentrated structure, which is fairly general as it contains oligopoly and monopoly as two special cases and is also aligned with the current practice in many aftermarkets. Our analysis shows that under MNL demand, the set of price equilibria coincides with the unique solutions to the system of first-order-condition equations. At equilibrium, the multi-contract firm prices its EWs via an equal-markup policy, whereas the equilibrium EW markups for the single-contract firms differ from each other. In this sense, the partially concentrated aftermarket behaves like a combination of monopoly and oligopoly. Comparing oligopoly, partial concentration, and monopoly demonstrates that a larger degree of market concentration softens competition, so that the EW prices become higher, and thus the total purchase probability and the consumer surplus become smaller. Even so, we find that the purchase probability of some contract in an oligopoly might be even lower than that in a monopoly, because of an interesting *market-share redistribution* effect induced by oligopolistic competition.

We then accommodate the ancillary nature of EWs by explicitly modeling their market potential (i.e., the product's sales volume) as a function of the product price determined by the manufacturer. On this basis, we study price competition for the main product and its EWs in which their prices are characterized by a sequential equilibrium. We find that when offering EWs in addition to the main product, the manufacturer would strategically reduce the product price to cultivate a larger aftermarket for EW sales. When the manufacturer, as the market leader, makes EW pricing decision before the single-contract firms, a price concession occurs in the aftermarket, thereby mitigating the intensity of competition. Even though it is non-trivial to compare the manufacturer's EW profits under the two sequential games in an analytical manner, our numerical result is consistent with the conventional wisdom—that is, compared to the simultaneous game, moving before the single-contract firms leads to a slightly higher per-unit EW profit for the manufacturer. This further translates to a lower product price, a larger product sales volume, and higher total profits for all firms. The numerical result advocates the simultaneous retail strategy (e.g., by posting the EW information together with the durable product), generalizing the insights in Heese (2012) and Wang et al. (2024) to a price competition scenario.

Our work exhibits several limitations that deserve future research efforts. Below we discuss some important ones and point out possible paths to address them.

(i) We consider symmetric price sensitivities—normalized to unity—in utility function (1), which results in the equal-markup pricing policy in the monopoly and the concentrated part of our concerned aftermarket. Generalizing to asymmetric price sensitivities would invalidate the equal-markup pricing policy, making it necessary to study price competition in various market scenarios with asymmetric price sensitivities. In doing so, the techniques proposed by Gallego and Wang (2014) and Li and Huh (2011) can be adopted to derive equilibrium outcomes.

(ii) A key feature of durable products absent from our analysis is the presence of second-hand markets, where pre-owned products can be transacted (Waldman, 2003). Studying price optimization and competition for EWs in the presence of a second-hand market is an interesting topic. The multi-period durable goods framework (see, e.g., Bulow, 1982; Huang et al., 2001) and the warranty compensation model (Utaka, 2006) can be adopted to formulate this problem. In particular, the finitely durable goods model developed by Huang et al. (2001) is helpful for characterizing the lengths of EWs. That is, EWs cover multiple periods and at the end of each period, consumers can sell their products in the second-hand market and, if allowed, transfer non-expired EW contracts to subsequent buyers (Zhang & Gao, 2021).

(iii) Finally, it is valuable to calibrate our demand model using real market data, which is also helpful for explaining the demand drivers of EWs. While Abito and Salant (2019) and Jindal

(2015) have conducted such empirical studies, it is of importance to base the studies on real market data derived from equilibrium outcomes in competitive environments. In other words, the demand model for EWs should be capable of accounting for the endogeneity of marketing decisions on EW prices and beyond. Chu and Chintagunta (2009) and Guajardo et al. (2016) have done similar investigations for base warranties, yet this type of empirical research for EWs is absent in the literature.

CRediT authorship contribution statement

Xiao-Lin Wang: Writing – original draft, Software, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Shizhe Peng:** Writing – review & editing, Validation, Conceptualization. **Xiaoge Zhang:** Writing – review & editing, Methodology, Conceptualization.

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Appendix A. Formulation of gross valuation v_i

We formulate the gross valuation v_i for each EW contract i by adapting the valuation model developed by Wang et al. (2020). To start with, suppose that the protection length of EW contract $i \in \mathcal{N}$ is W_i, so the associated protection period is $[W_b, W_b + W_i]$. As EWs might not cover all major components of the durable product, let $\mathcal{S}_i$ represent the set of components covered by EW contract i.

Now consider each component $k \in \mathcal{S}_i$ covered by EW contract i. In essence, the valuation of EW i for component k realizes when this component fails in the protection period $[W_b, W_b + W_i]$. Let $F_{i,k}$ denote the probability that component k fails in $[W_b, W_b + W_i]$, and $d_{i,k}$ represent the amount of compensation received by consumers if this failure event occurs. That is, if component k fails over the EW period, then the consumers would receive compensation $d_{i,k}$; otherwise, the EW contract becomes valueless. Taking into account the component's failure probability, the average valuation of EW i for component k is $F_{i,k} \cdot d_{i,k} + (1 - F_{i,k}) \cdot 0 = d_{i,k} \cdot F_{i,k}$.

Because the set of components covered by EW i is $\mathcal{S}_i$, the total valuation EW i brings to representative consumers is given by

$$v_i = \sum_{k \in \mathcal{S}_i} d_{i,k} \cdot F_{i,k}. \tag{A.1}$$

The failure probability $F_{i,k}$ is formulated as $F_{i,k} = \int_{W_b}^{W_b+W_i} \lambda_k(t) \mathrm{d}t$, where $\lambda_k(t)$ is the hazard rate function of component k.

Appendix B. Solving the equilibrium for the simultaneous game

Following the discussion in Section 3.2, we develop a simple bisection-type algorithm for computing the equilibrium in Theorem 1. The inputs of this algorithm include a lower bound $q_{0,l}$ and an upper bound $q_{0,u}$ for the no-purchase probability q_0^*, as well as an error-tolerance $\varepsilon > 0$.

Appendix C. Solving the equilibrium for the sequential game II

Solving the price equilibria for the sequential game II can be done in a fairly similar manner to that for the simultaneous game presented

Algorithm 1 Computing the equilibrium prices.

Input: $q_{0,l}$, $q_{0,u}$, ε
Output: $p^* = (p_1^*, p_2^*, \ldots, p_n^*)$
1: Compute $q_{0,m} = (q_{0,l} + q_{0,u})/2$
2: Obtain $\Phi_i(q_{0,m})$ by solving $q_{0,m} = f_i(q_i)$ for q_i, $\forall i \in \mathcal{N}_{k^-}$
3: Obtain $\check{\Phi}_i(q_{0,m})$ by solving $q_{0,m} = \check{f}_i(q_i)$ for q_i, $\forall i \in \mathcal{N}_k$
4: Compute $Q_m = \sum_{i \in \mathcal{N}_{k^-}} \Phi_i(q_{0,m}) + \sum_{i \in \mathcal{N}_k} \check{\Phi}_i(q_{0,m}) + q_{0,m}$
5: **while** $|Q_m - 1| \geq \varepsilon$ **do**
6:   **if** $Q_m - 1 > 0$ **then**
7:     $q_{0,u} \leftarrow q_{0,m}$
8:   **else**
9:     $q_{0,l} \leftarrow q_{0,m}$
10:   **end if**
11:   Repeat Steps 1-4
12: **end while**
13: Set $q_i^* \leftarrow \Phi_i(q_{0,m})$, $\forall i \in \mathcal{N}_{k^-}$
14: Set $q_i^* \leftarrow \check{\Phi}_i(q_{0,m})$, $\forall i \in \mathcal{N}_k$
15: Compute p_i^*, $\forall i \in \mathcal{N}$, by Theorem 1
16: **return** p^*

in Section 3.2. According to the proof of Proposition 3, $\hat{\Phi}_i(q_0)$, $i \in \mathcal{N}_k$, is the unique solution to $q_0 = \hat{f}_i(q_i)$ for any given $q_0 \in (0,1)$, where $\hat{f}_i(q_i) := \frac{q_i}{A_i} \exp\{\frac{1}{1 - \frac{1}{1+\Theta}\sum_{j \in \mathcal{N}_k} q_j}\}$. Then, at equilibrium, we must have

$$\sum_{i \in \mathcal{N}_{k^-}} \Phi_i(q_0) + \sum_{i \in \mathcal{N}_k} \hat{\Phi}_i(q_0) + q_0 = 1.$$

It is not difficult to verify that there exists a unique $\hat{q}_0^* \in (0,1)$ that satisfies this equality.

However, the above procedure is not computation-friendly, because the function $\hat{f}_i(q_i)$ depends not only on q_i, but also on q_j ($\forall j \in \mathcal{N}_k \setminus \{i\}$) and $\Phi_j(q_0)$ ($\forall j \in \mathcal{N}_{k^-}$). In fact, we can transform the $|\mathcal{N}_k|$ root-finding problems (i.e., $q_0 = \hat{f}_i(q_i)$, $\forall i \in \mathcal{N}_k$) into a single one. For ease of presentation, let $Q_k = \sum_{i \in \mathcal{N}_k} q_i$. First notice—from $q_0 = \hat{f}_i(q_i)$—that $q_i = A_i \exp\{-\frac{1}{1 - \frac{1}{1+\Theta}Q_k}\} q_0$, $\forall i \in \mathcal{N}_k$. Then, we have $Q_k = \sum_{i \in \mathcal{N}_k} q_i = \exp\{-\frac{1}{1-\frac{1}{1+\Theta}Q_k}\} q_0 \sum_{i \in \mathcal{N}_k} A_i$. This leads to $q_0 = Q_k \exp\{\frac{1}{1-\frac{1}{1+\Theta}Q_k}\} \frac{1}{\sum_{i \in \mathcal{N}_k} A_i}$ and thus

$$q_i = Q_k \frac{A_i}{\sum_{j \in \mathcal{N}_k} A_j}. \tag{C.1}$$

Substituting (C.1) into $q_0 = \hat{f}_i(q_i)$ yields

$$q_0 = \frac{Q_k}{\sum_{i \in \mathcal{N}_k} A_i} \exp\left\{\frac{1}{1 - \frac{1}{1+\Theta} Q_k}\right\},$$

which has a unique solution $\hat{Q}_k(q_0) \in (0, 1+\Theta)$ for any $q_0 \in (0,1)$. Then, the equilibrium $\hat{q}_0^*$ can be obtained by solving

$$\sum_{i \in \mathcal{N}_{k^-}} \Phi_i(q_0) + \hat{Q}_k(q_0) + q_0 = 1.$$

Therefore, from the computational perspective, once a unique $\hat{Q}_k(\hat{q}_0^*)$ is determined for a given $\hat{q}_0^*$, $\hat{q}_i^*$ for each $i \in \mathcal{N}_k$ can be obtained by (C.1). This requires solving only one root-finding problem for a given $\hat{q}_0^*$, which is much easier to handle.

Finally, we note that this method also applies to Step 3 in Algorithm 1, in which $\check{\Phi}_i(q_{0,m})$ is computed by solving $q_{0,m} = \check{f}_i(q_i)$ for q_i, $\forall i \in \mathcal{N}_k$. This is because the function $\check{f}_i(q_i)$ depends not only on q_i, but also on q_j ($\forall j \in \mathcal{N}_k \setminus \{i\}$).

Appendix D. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ejor.2025.04.001.

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